

Review 238

To add or subtract fractions and mixed numbers with unlike denominators, first rewrite the fractions using the least common denominator (LCD).

Subtract: $2\frac{3}{4} - 5\frac{1}{3}$

$$\begin{aligned} 2\frac{3}{4} - 5\frac{1}{3} &= \frac{11}{4} - \frac{16}{3} \\ &= \frac{33}{12} - \frac{64}{12} \quad \leftarrow \text{The LCD is 12.} \\ &= \frac{-31}{12} \quad \leftarrow \text{Subtract numerators.} \\ &= -2\frac{7}{12} \quad \leftarrow \text{Simplify.} \end{aligned}$$

$$2\frac{3}{4} - 5\frac{1}{3} = -2\frac{7}{12}$$

Adding and Subtracting Rational Numbers

You can use addition or subtraction to solve equations with rational numbers.

Solve: $h - \frac{3}{8} = \frac{1}{6}$

$$\begin{aligned} h - \frac{3}{8} + \frac{3}{8} &= \frac{1}{6} + \frac{3}{8} \quad \leftarrow \text{Add } \frac{3}{8}. \\ h &= \frac{4}{24} + \frac{9}{24} \quad \leftarrow \text{The LCD is 24.} \\ h &= \frac{13}{24} \end{aligned}$$

Find each sum or difference as a fraction or mixed number in simplest form.

1. $6\frac{1}{4} - 2\frac{3}{8}$

$$\underline{5\frac{10}{8} - 2\frac{3}{8}} = 3\frac{7}{8}$$

2. $\frac{5}{6} + \left(-\frac{1}{2}\right)$

$$\underline{\frac{5}{6} + \left(-\frac{3}{6}\right)} = \frac{2}{6} = \frac{1}{3}$$

3. $-4\frac{1}{3} - \left(-\frac{3}{5}\right)$

$$\underline{-3\frac{20}{15} + \frac{9}{15}} = -3\frac{11}{15}$$

4. $\frac{1}{8} - \left(-\frac{1}{6}\right)$

$$\underline{\frac{3}{24} + \frac{4}{24}} = \frac{7}{24}$$

5. $-1\frac{3}{8} - 4\frac{1}{12}$

$$\underline{-1\frac{9}{24} + \left(-4\frac{2}{24}\right)} = -5\frac{11}{24}$$

6. $\frac{7}{10} + \left(-1\frac{2}{5}\right)$

$$\underline{\frac{7}{10} + \left(-\frac{14}{10}\right)} = -\frac{7}{10}$$

7. $1\frac{5}{8} - \left(-2\frac{1}{2}\right)$

$$\underline{1\frac{5}{8} + 2\frac{4}{8}} = 3\frac{9}{8} = 4\frac{1}{8}$$

8. $-2\frac{1}{3} - \left(-1\frac{5}{12}\right)$

$$\underline{-2\frac{4}{12} + 1\frac{5}{12}} = -\frac{11}{12}$$

9. $-10 - \left(3\frac{11}{12}\right)$

$$\underline{-10 - 3\frac{11}{12}} = -13\frac{11}{12}$$

10. $1\frac{1}{3} - 4\frac{3}{4}$

$$\underline{1\frac{4}{12} - 4\frac{9}{12}} = -3\frac{5}{12}$$

11. $9 + \left(-6\frac{5}{9}\right)$

$$\underline{8\frac{9}{9} + \left(-6\frac{5}{9}\right)} = 2\frac{4}{9}$$

12. $-2\frac{5}{6} - 5\frac{5}{12}$

$$\underline{-2\frac{10}{12} - 5\frac{5}{12}} = -7\frac{15}{12} = -8\frac{1}{4}$$

Solve each equation. Write each answer as a mixed number or as a fraction in simplest form.

13. $y + \frac{7}{8} = -\frac{1}{4}$

$$\underline{-\frac{7}{8} - \frac{7}{8}} \quad y = -\frac{9}{8} = -1\frac{1}{8}$$

14. $c + \frac{3}{5} = \frac{1}{2}$

$$\underline{+\frac{3}{5} - \frac{3}{5}} \quad c = \frac{1}{10} = 1\frac{1}{10}$$

15. $m - 3\frac{2}{3} = 1\frac{1}{6}$

$$\underline{+3\frac{2}{3} + 3\frac{2}{3}} \quad m = 4\frac{5}{6}$$

16. $x - 2\frac{1}{4} = -3$

$$\underline{+2\frac{1}{4} + 2\frac{1}{4}} \quad x = -\frac{3}{4}$$

17. $n + \frac{1}{2} = -2\frac{5}{6}$

$$\underline{-\frac{1}{2} - \frac{1}{2}} \quad n = -3\frac{1}{3}$$

18. $\frac{1}{2} + d = -3\frac{1}{5}$

$$\underline{d = -3\frac{7}{10}}$$

19. $7.3 + g = 1\frac{4}{5}$

$$\underline{-7\frac{3}{10} - 7\frac{3}{10}} \quad g = -5\frac{1}{2}$$

20. $y - 4.1 = 2\frac{3}{4}$

$$\underline{y = 6\frac{17}{20}}$$

21. $z + 2.6 = 0.37$

$$\underline{-2.6 - 2.6} \quad z = 0.23$$

$$\underline{-2\frac{23}{100}}$$

Review 239**Multiplying and Dividing Rational Numbers**

To multiply rational numbers in fraction form, multiply numerators, then multiply denominators.

Multiply: $\frac{7}{12} \cdot 1\frac{4}{5}$

$$\begin{array}{r} \frac{7}{12} \cdot \frac{9}{5} \\ \text{← fraction form} \\ \frac{7 \cdot 9}{12 \cdot 5} \\ \text{← Multiply numerators.} \\ \text{← Multiply denominators.} \end{array}$$

$$\frac{63}{60} = 1\frac{3}{60} = 1\frac{1}{20} \quad \text{← Simplify.}$$

To divide, multiply by the reciprocal of the divisor.

Divide: $-3\frac{1}{8} \div \frac{2}{3}$

$$\begin{array}{r} \frac{-25}{8} \div \frac{2}{3} \\ \text{← fraction form} \\ \frac{-25}{8} \cdot \frac{3}{2} \\ \text{← reciprocal of divisor} \\ \frac{-25 \cdot 3}{8 \cdot 2} = \frac{-75}{16} \quad \text{← Multiply.} \\ = -4\frac{11}{16} \quad \text{← Simplify.} \end{array}$$

Find each product. Write each answer as a fraction or mixed number in simplest form.

1. $\frac{2}{3} \cdot \left(\frac{3}{4}\right)$

$$\begin{array}{r} \frac{2}{3} \cdot \frac{3}{4} \\ \text{---} \\ -\frac{2}{3} \end{array}$$

2. $-\frac{1}{2} \cdot \frac{4}{5}$

$$\begin{array}{r} -\frac{1}{2} \cdot \frac{4}{5} \\ \text{---} \\ -\frac{2}{5} \end{array}$$

3. $-\frac{2}{3} \cdot \left(-\frac{1}{8}\right)$

$$\begin{array}{r} -\frac{2}{3} \cdot -\frac{1}{8} \\ \text{---} \\ \frac{1}{12} \end{array}$$

4. $\frac{5}{6} \cdot \frac{5}{7}$

$$\begin{array}{r} \frac{5}{6} \cdot \frac{5}{7} \\ \text{---} \\ \frac{5}{14} \end{array}$$

5. $\frac{3}{4} \cdot \left(-\frac{2}{3}\right)$

$$\begin{array}{r} \frac{3}{4} \cdot -\frac{2}{3} \\ \text{---} \\ -\frac{1}{2} \end{array}$$

6. $3 \cdot 2\frac{1}{4}$

$$\begin{array}{r} 3 \cdot 2\frac{1}{4} \\ \frac{3}{1} \times \frac{9}{4} \\ \text{---} \\ \frac{27}{4} = 6\frac{3}{4} \end{array}$$

7. $-5\frac{1}{2} \cdot 1\frac{3}{4}$

$$\begin{array}{r} -\frac{11}{2} \cdot \frac{7}{4} = -\frac{77}{8} = -9\frac{5}{8} \end{array}$$

8. $-2\frac{1}{8} \cdot (-3)$

$$\begin{array}{r} -\frac{17}{8} \cdot (-3) \\ \frac{51}{8} = 6\frac{3}{8} \end{array}$$

9. $4\frac{1}{5} \cdot 2\frac{1}{2}$

$$\begin{array}{r} \frac{21}{5} \cdot \frac{5}{2} = \frac{21}{2} = 10\frac{1}{2} \end{array}$$

10. $\frac{13}{18} \cdot \frac{5}{6}$

$$\begin{array}{r} \frac{13}{18} \cdot \frac{5}{6} \\ \text{---} \\ \frac{13}{18} \end{array}$$

11. $-3\frac{2}{5} \cdot 2\frac{1}{2}$

$$\begin{array}{r} -\frac{17}{5} \cdot \frac{5}{2} = -\frac{17}{2} = -8\frac{1}{2} \end{array}$$

12. $-5 \cdot \left(-2\frac{1}{4}\right)$

$$\begin{array}{r} -5 \cdot \left(-\frac{9}{4}\right) = \frac{45}{4} = 11\frac{1}{4} \end{array}$$

13. $-\frac{5}{8} \cdot 4\frac{2}{3}$

$$\begin{array}{r} -\frac{5}{8} \cdot \frac{14}{3} = -\frac{35}{12} = -2\frac{11}{12} \end{array}$$

14. $-5 \cdot 3\frac{3}{10}$

$$\begin{array}{r} -5 \cdot \left(\frac{33}{10}\right) = -\frac{33}{2} = -16\frac{1}{2} \end{array}$$

15. $-2\frac{3}{5} \cdot \left(-3\frac{1}{3}\right)$

$$\begin{array}{r} -\frac{13}{5} \cdot \left(-\frac{10}{3}\right) = \frac{130}{15} = 8\frac{2}{3} \end{array}$$

Find each quotient.

16. $\frac{5}{6} \div \frac{3}{5}$

$$\begin{array}{r} \frac{5}{6} \times \frac{5}{3} = \frac{25}{18} = 1\frac{7}{18} \end{array}$$

17. $-\frac{3}{8} \div \left(-\frac{1}{2}\right)$

$$\begin{array}{r} -\frac{3}{8} \times \left(-\frac{4}{1}\right) = \frac{3}{4} \end{array}$$

18. $-6 \div \frac{3}{4}$

$$\begin{array}{r} -6 \cdot \frac{4}{3} = -8 \end{array}$$

19. $4 \div \left(-\frac{2}{3}\right)$

$$\begin{array}{r} 4 \times \left(-\frac{3}{2}\right) = -6 \end{array}$$

20. $5\frac{1}{4} \div 1\frac{1}{2}$

$$\begin{array}{r} \frac{21}{4} \cdot \frac{2}{3} = \frac{7}{2} = 3\frac{1}{2} \end{array}$$

21. $1\frac{1}{4} \div \left(-\frac{2}{5}\right)$

$$\begin{array}{r} \frac{5}{4} \times \left(-\frac{5}{2}\right) = -\frac{25}{8} = -3\frac{1}{8} \end{array}$$

22. $-\frac{3}{4} \div \left(-1\frac{1}{2}\right)$

$$\begin{array}{r} -\frac{3}{4} \times \left(-\frac{2}{3}\right) = \frac{1}{2} \end{array}$$

23. $-1\frac{3}{5} \div \frac{1}{4}$

$$\begin{array}{r} -\frac{8}{5} \times 4 = -\frac{32}{5} = -6\frac{2}{5} \end{array}$$

24. $2\frac{1}{2} \div \frac{3}{10}$

$$\begin{array}{r} \frac{5}{2} \times \frac{10}{3} = \frac{50}{6} = 8\frac{1}{3} \end{array}$$

25. $-\frac{5}{9} \div \left(-\frac{2}{3}\right)$

$$\begin{array}{r} -\frac{5}{9} \times \left(-\frac{3}{2}\right) = \frac{5}{6} \end{array}$$

26. $-6 \div 3\frac{5}{8}$

$$\begin{array}{r} -6 \times \frac{8}{29} = -\frac{48}{29} = -1\frac{19}{29} \end{array}$$

27. $\frac{3}{4} \div (-9)$

$$\begin{array}{r} \frac{3}{4} \times \left(-\frac{1}{9}\right) = -\frac{1}{12} \end{array}$$

Review 242

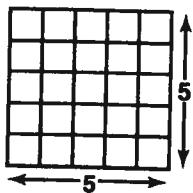
Exploring Square Roots and Irrational Numbers

The square of 5 is 25.

$$5 \cdot 5 = 5^2 = 25$$

The *square root* of 25 is 5 because $5^2 = 25$.

$$\sqrt{25} = 5$$



$$\left. \begin{array}{l} 1^2 = 1 \\ 2^2 = 4 \\ 3^2 = 9 \\ 4^2 = 16 \\ 5^2 = 25 \end{array} \right\} \text{perfect squares}$$

You can use a calculator to find square roots.

Example: Find $\sqrt{36}$ and $\sqrt{21}$ to the nearest tenth.

$$36 \boxed{} = 6 \quad 21 \boxed{} \approx 4.5825757 \approx 4.6$$

You can estimate square roots like $\sqrt{52}$ and $\sqrt{61}$.

Perfect squares	49	$\sqrt{49} = 7$	$\sqrt{49} = 7$
	52	Estimate	$\sqrt{52} \approx 7$
	64		$\sqrt{61} \approx 8$

Perfect squares	$\sqrt{64} = 8$	$\sqrt{61} \approx 8$
		$\sqrt{64} = 8$

Find each square root. Round to the nearest integer if necessary.

Use \approx to show that a value is rounded.

- | | | | |
|--------------------------------|-------------------------------|--------------------------------|--------------------------------|
| 1. $\sqrt{16}$ | 2. $\sqrt{85}$ | 3. $\sqrt{26}$ | 4. $\sqrt{36}$ |
| <u>4</u> | <u>≈ 9</u> | <u>≈ 5</u> | <u>6</u> |
| 5. $\sqrt{98}$ | 6. $\sqrt{40}$ | 7. $\sqrt{100}$ | 8. $\sqrt{18}$ |
| <u>≈ 10</u> | <u>≈ 6</u> | <u>10</u> | <u>≈ 4</u> |
| 9. $\sqrt{5}$ | 10. $\sqrt{121}$ | 11. $\sqrt{68}$ | 12. $\sqrt{144}$ |
| <u>≈ 2</u> | <u>11</u> | <u>≈ 8</u> | <u>12</u> |
| 13. $\sqrt{29}$ | 14. $\sqrt{64}$ | 15. $\sqrt{37}$ | 16. $\sqrt{75}$ |
| <u>≈ 5</u> | <u>8</u> | <u>≈ 6</u> | <u>≈ 9</u> |
| 17. $\sqrt{225}$ | 18. $\sqrt{54}$ | 19. $\sqrt{169}$ | 20. $\sqrt{103}$ |
| <u>15</u> | <u>≈ 7</u> | <u>13</u> | <u>≈ 10</u> |
| 21. $\sqrt{61}$ | 22. $\sqrt{400}$ | 23. $\sqrt{119}$ | 24. $\sqrt{84}$ |
| <u>≈ 8</u> | <u>20</u> | <u>≈ 11</u> | <u>≈ 9</u> |

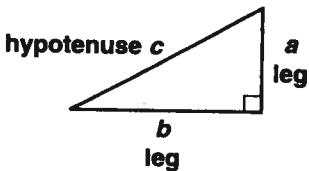
25. If a whole number is not a perfect square, its square root is an *irrational number*. List the numbers from Exercises 1–24 that are irrational.

$\sqrt{85}$, $\sqrt{26}$, $\sqrt{98}$, $\sqrt{40}$, $\sqrt{18}$, $\sqrt{5}$, $\sqrt{68}$, $\sqrt{29}$, $\sqrt{37}$,
 $\sqrt{75}$, $\sqrt{54}$, $\sqrt{103}$, $\sqrt{61}$, $\sqrt{119}$, $\sqrt{84}$

Review 243**The Pythagorean Theorem*****The Pythagorean Theorem***

The sum of the squares of the lengths of the *legs* of a right triangle is equal to the square of the length of the *hypotenuse*.

Also, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.



$$a^2 + b^2 = c^2$$

Example 1: Find the length of a leg of a right triangle if the length of the other leg is 12 cm and the length of the hypotenuse is 13 cm.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 12^2 + b^2 &= 13^2 \\ 144 + b^2 &= 169 \\ 144 - 144 + b^2 &= 169 - 144 \\ b^2 &= 25 \\ b &= \sqrt{25} \\ b &= 5 \end{aligned}$$

The length of the leg is 5 cm.

Example 2: Is a triangle with sides 6 m, 7 m, and 10 m a right triangle?

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 7^2 &\stackrel{?}{=} 10^2 && \leftarrow \text{Substitute.} \\ 36 + 49 &\stackrel{?}{=} 100 && \leftarrow \text{Simplify.} \\ 85 &\neq 100 \end{aligned}$$

The triangle is *not* a right triangle.

The lengths of two sides of a right triangle are given. Find the length of the third side.

1. legs: 6 ft and 8 ft
hypotenuse:

10ft

2. leg: 15 m
hypotenuse: 17 m
leg:

8m

3. leg: 12 in.
hypotenuse: 15 in.
leg:

9in

4. leg: 1.5 km
hypotenuse: 2.5 km
leg:

2km

5. legs: 15 in. and 20 in.
hypotenuse:

25in

6. leg: 16 m
hypotenuse: 34 m
leg:

30m

Is a triangle with the given side lengths a right triangle?

7. 10 cm, 24 cm, 26 cm

yes

8. 5 ft, 7 ft, 9 ft

no

9. 6 m, 12 m, 15 m

no

10. 5 in., 12 in., 13 in.

yes

11. 30 mm, 40 mm, 50 mm

yes

12. 2 yd, 5 yd, 8 yd

no

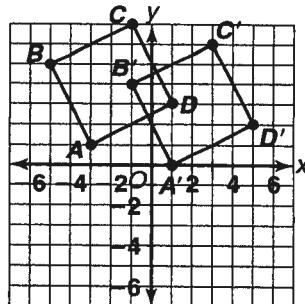
Review 191**Translations**

Movements of figures on a plane are called **transformations**. A translation, or slide, moves all points the same distance and direction.

The translation $(x, y) \rightarrow (x + 4, y - 1)$ moves each point to the right 4 units and down 1 unit.

$A(-3, 1)$ moves to $(-3 + 4, 1 - 1)$, where point $A'(1, 0)$ is its **image**.

The square $ABCD$ moves to its image square $A'B'C'D'$.



Complete the following for the figure above.

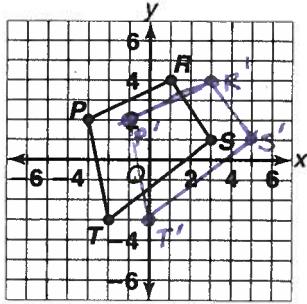
1. $B(-5, 5) \rightarrow B'(-1, 4)$

2. $C(-1, 7) \rightarrow C'(3, 6)$

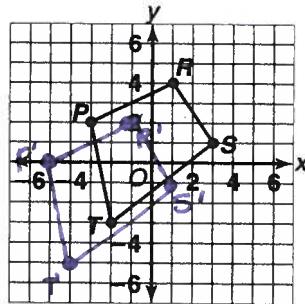
3. $D(1, 3) \rightarrow D'(5, 2)$

Graph each translation of figure $PRST$.

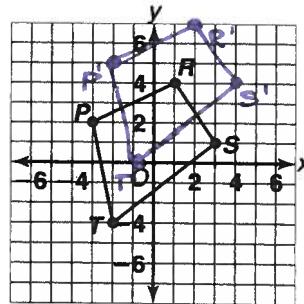
4. right 2 units



5. left 2 units, down 2 units



6. right 1 unit, up 3 units



Complete the rule for each translation.

7. right 3 units, up 1 unit

$$(x, y) \rightarrow (x+3, y+1)$$

8. left 4 units, up 5 units

$$(x, y) \rightarrow (x-4, y+5)$$

9. left 1 unit, down 9 units

$$(x, y) \rightarrow (x-1, y-9)$$

Write a rule for the translation.

10. left 1 unit, down 3 units

$$(x, y) \rightarrow (x-1, y-3)$$

11. right 1 unit, up 2 units

$$(x, y) \rightarrow (x+1, y+2)$$

12. left 3 units, up 2 units

$$(x, y) \rightarrow (x-3, y+2)$$

Review 232**Translations**

A **translation** moves every point of a figure the same distance in the same direction.

Triangle ABC is translated 5 units to the right and 4 units up. The **image** of $\triangle ABC$ is $\triangle A'B'C'$.

You can write a rule to describe a translation in the coordinate plane.

For the translation of $\triangle DEF$, the rule is:

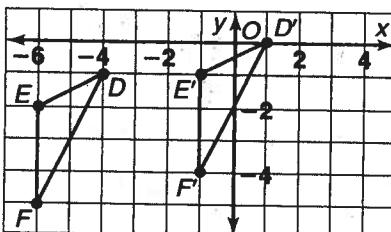
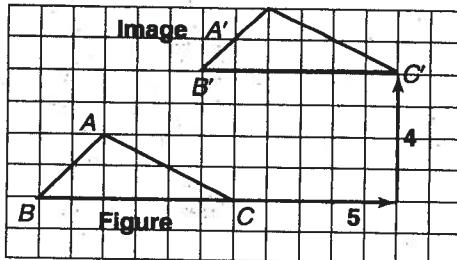
Add 5 to each x -coordinate.

Add 1 to each y -coordinate.

$$D(-4, -1) \rightarrow D'(1, 0)$$

$$E(-6, -2) \rightarrow E'(-1, -1)$$

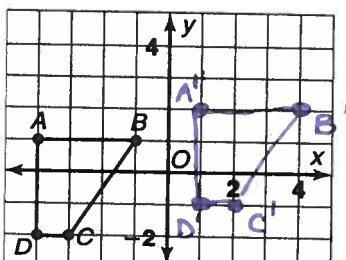
$$F(-6, -5) \rightarrow F'(-1, -4)$$



Copy each figure. Then graph the image after the given translation.

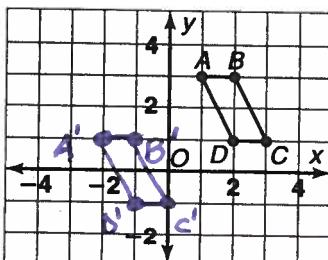
Name the coordinates of the image.

1. right 5 units, up 1 unit



$$\begin{aligned} A'(3, 0), B'(5, 0) \\ C'(3, -1), D'(1, -1) \end{aligned}$$

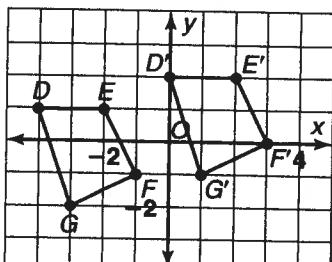
2. left 3 units, down 2 units



$$\begin{aligned} A'(-1, 1), B'(-2, 1) \\ C'(-3, 0), D'(-1, -1) \end{aligned}$$

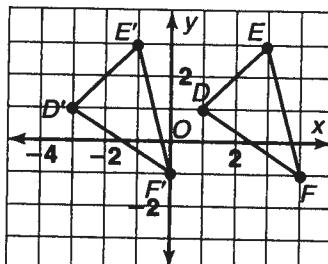
Use arrow notation to write a rule that describes the translation shown on each graph.

3.



$$(x, y) \rightarrow (x + 4, y + 1)$$

4.



$$(x, y) \rightarrow (x - 4, y + 2)$$

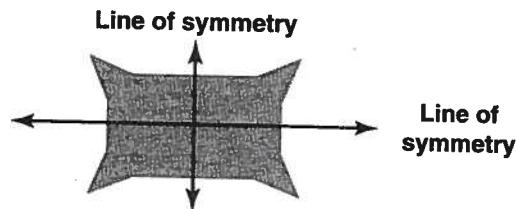
Review 192

Symmetry and Reflections

Symmetry

A figure is **symmetrical** if one side is a mirror image of the other. The line that divides a figure into two identical parts is called a **line of symmetry**.

The figure below has 2 lines of symmetry.

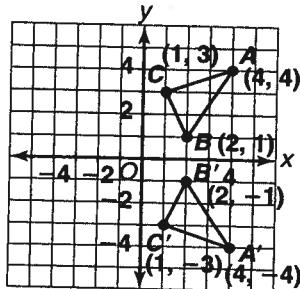


Course 2 Topics

You can trace the figure and fold it along either line to see that the two halves match.

Reflections

A **reflection** is a transformation that creates a mirror image. $\triangle A'B'C'$ is the mirror image of $\triangle ABC$ across the x -axis. The x -axis is the **line of reflection**.



- When you reflect across the x -axis, the y -coordinates change sign.
- When you reflect across the y -axis, the x -coordinates change sign.
- When you reflect across a line of symmetry, the image is the figure itself.

Draw the line(s) of symmetry. If there are no lines of symmetry, write *none*.

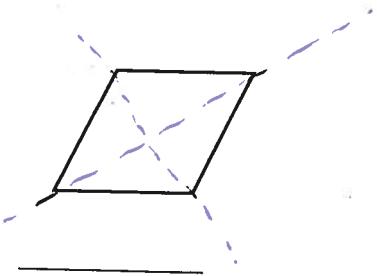
1.



2.



3.



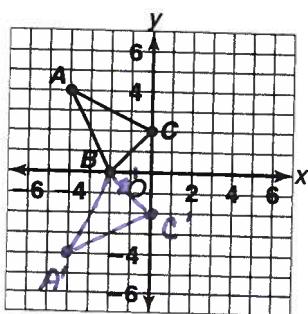
$\triangle ABC$ is shown. Draw $\triangle A'B'C'$ so it is a reflection of $\triangle ABC$ over the specified axis. Then complete each statement.

4. over the x -axis

$$A(-4, 4) \rightarrow A' \underline{(-4, -4)}$$

$$B(-2, 0) \rightarrow B' \underline{(-2, 0)}$$

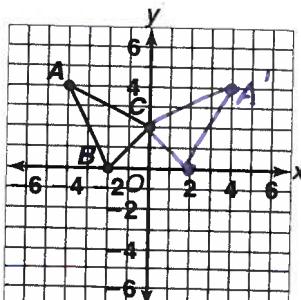
$$C(0, 2) \rightarrow C' \underline{(0, -2)}$$

5. over the y -axis

$$A(-4, 4) \rightarrow A' \underline{(4, 4)}$$

$$B(-2, 0) \rightarrow B' \underline{(2, 0)}$$

$$C(0, 2) \rightarrow C' \underline{(0, 2)}$$



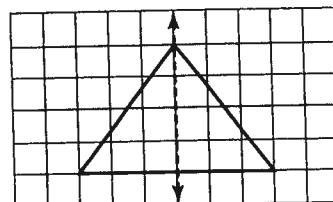
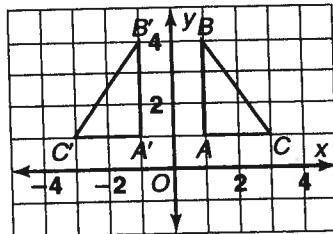
Reflections and Symmetry**Review 233**

A *reflection* flips a figure over a line (the *line of reflection*). Figure $A'B'C'$ is the image of figure ABC after a reflection over the y -axis.

Each point of the image is the same distance from the line of reflection as the corresponding point of the original figure.

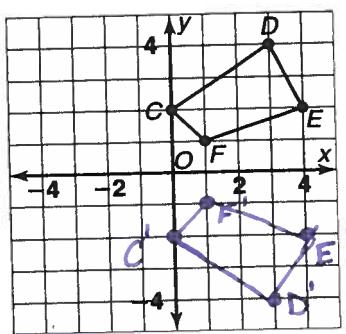
Since A is 1 unit to the right of the y -axis, locate A' 1 unit to the left of the y -axis.

If the image is identical to the original figure, then the figure has *reflectational symmetry* and has a *line of symmetry*.

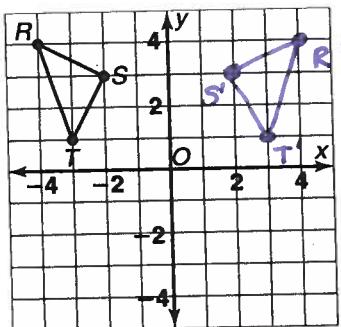


Copy each figure.

1. Reflect the figure over the x -axis.

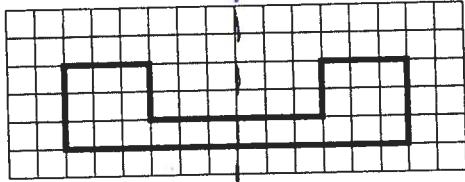


2. Reflect the figure over the y -axis.



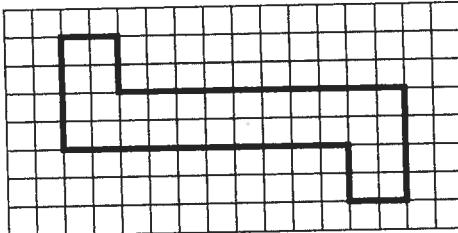
Copy each figure. Does the figure have reflectational symmetry? If it does, draw all the lines of symmetry.

3.



yes

4.



no

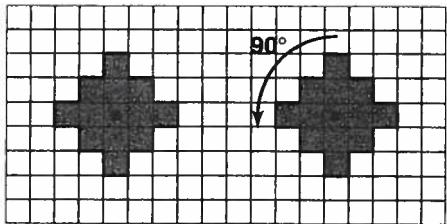
Review 193

Rotations

A **rotation** is a transformation that turns a figure about a fixed point. The fixed point is called the **center of rotation**.

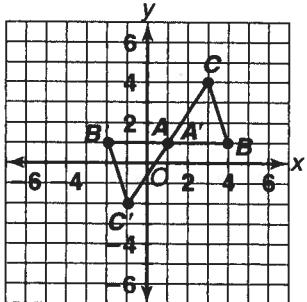
A figure has **rotational symmetry** if it can be rotated less than 360° and fit exactly on top of the original figure.

The figure below has rotational symmetry.



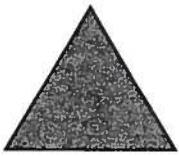
For a rotation of 90° or 180° about its center, the figure fits exactly on top of itself.

To draw a 180° rotation about point A , trace $\triangle ABC$. Place a pencil tip on point A and rotate the tracing 180° . Mark points A' , B' , and C' . Then draw $\triangle A'B'C'$.



Does each figure have rotational symmetry?

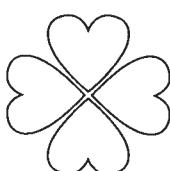
1.

yes

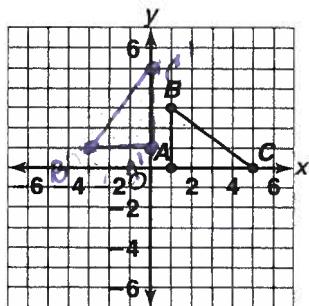
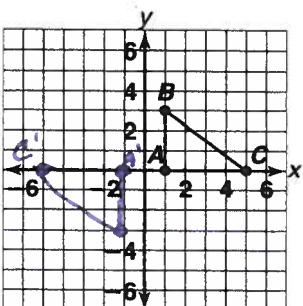
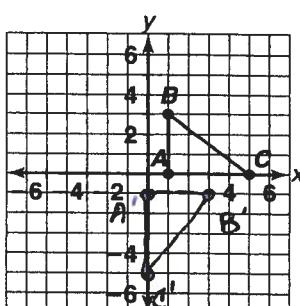
2.

no

3.

yes

Draw the image of the figure after each rotation about point O .

4. rotation of 90° 5. rotation of 180° 6. rotation of 270° 

Review 234**Rotations**

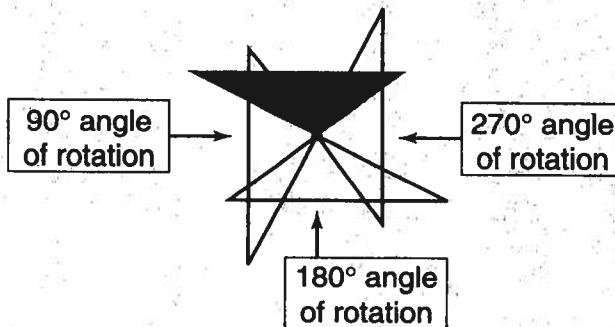
A **rotation** is a turn of a figure about a center point, the *center of rotation*.

A figure can be rotated up to 360° counterclockwise.

A figure has **rotational symmetry** if an image matches the original figure after a rotation of 180° or less.

The angle measure the figure rotates is the *angle of rotation*.

The shaded triangle is rotated about its lower vertex.

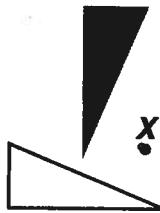


The triangle does not have rotational symmetry.

The shaded figure is rotated 90° , 180° , or 270° about point X.

The unshaded figure is its image. What is the angle of rotation?

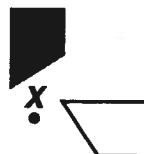
1.

 90°

2.

 180°

3.

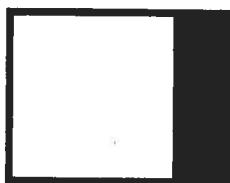
 270°

Judging by appearance, determine whether each figure has rotational symmetry. If it does, find the angle of rotation.

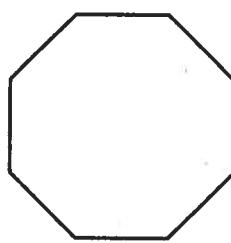
4.

 180°

5.

none

6.

 45°

Review 269**Angles and Parallel Lines**

Look at the figure at the right.

- Line \overleftrightarrow{AB} is parallel to line \overleftrightarrow{CD} ($\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)
- Line \overleftrightarrow{EF} is a *transversal*.

Alternate interior angles lie within a pair of lines and on opposite sides of the transversal.

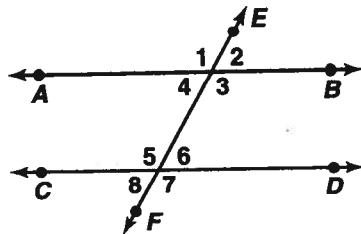
Example 1: $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

Alternate interior angles are congruent. If $m\angle 4$ is 60° , then $m\angle 6$ is also 60° .

Corresponding angles lie on the same side of the transversal and in corresponding positions.

Example 2: $\angle 1$ and $\angle 5$, $\angle 3$ and $\angle 7$

Corresponding angles are congruent. If $m\angle 1$ is 120° , then $m\angle 5$ is also 120° .



Use the diagram at the right to complete Exercises 1–2.

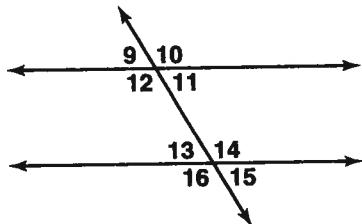
1. Name the alternate interior angles.

a. $\angle 11$ and $\angle ?$

$\angle 13$

b. $\angle 12$ and $\angle ?$

$\angle 14$



2. Name the corresponding angles.

a. $\angle 16$ and $\angle ?$

$\angle 12$

b. $\angle 14$ and $\angle ?$

$\angle 10$

c. $\angle 9$ and $\angle ?$

$\angle 13$

d. $\angle 11$ and $\angle ?$

$\angle 15$

In the diagram at the right, $\ell \parallel m$. Find the measure of each angle.

3. $\angle 1$

150°

4. $\angle 3$

150°

5. $\angle 6$

30°

6. $\angle 5$

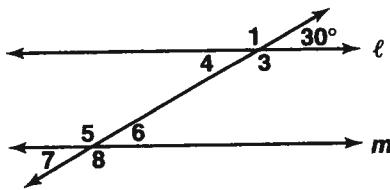
150°

7. $\angle 8$

150°

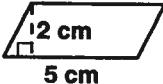
8. $\angle 7$

30°



Review 274**Areas of Polygons**

Example 1: Find the area of the parallelogram. Use the formula below.

$$\text{Area} = \text{base} \times \text{height}$$


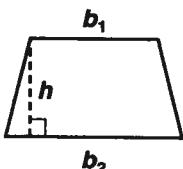
$$A = bh$$

$$= 5 \times 2$$

$$= 10 \text{ cm}^2$$

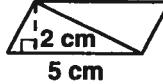
The area of a trapezoid is half the product of the height and the sum of the lengths of the bases.

$$A = \frac{1}{2}h(b_1 + b_2)$$



Example 2: Find the area of the triangle. You can cut a parallelogram into two congruent triangles. So, the area of a triangle is half the area of a parallelogram.

To find the area of a triangle, use this formula.

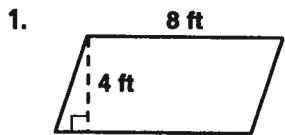
$$\text{Area} = \frac{1}{2}\text{base} \times \text{height}$$


$$A = \frac{1}{2}bh$$

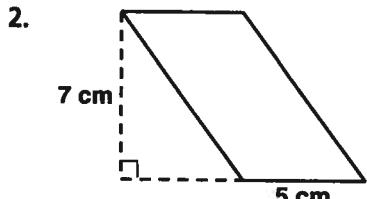
$$= \frac{1}{2} \times 5 \times 2$$

$$= 5 \text{ cm}^2$$

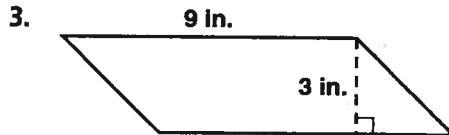
Find the area of each parallelogram.



$$A = 32 \text{ ft}^2$$

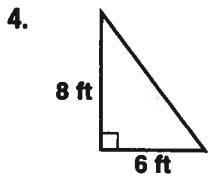


$$A = 35 \text{ cm}^2$$

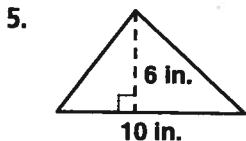


$$A = 27 \text{ in.}^2$$

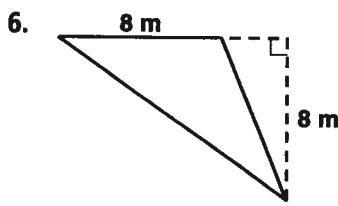
Find the area of each triangle.



$$A = 24 \text{ ft}^2$$

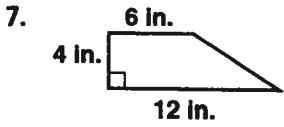


$$A = 30 \text{ in.}^2$$

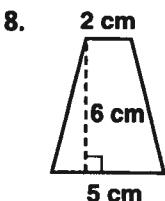


$$A = 32 \text{ m}^2$$

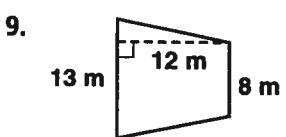
Find the area of each trapezoid.



$$A = 36 \text{ in.}^2$$



$$A = 21 \text{ cm}^2$$



$$A = 126 \text{ m}^2$$

Review 275**Circumferences and Areas of Circles**

The distance around a circle is called the *circumference*.

- You can use a formula to find the circumference (C) of a circle. $\text{Pi } (\pi)$ is approximately equal to (\approx) 3.14.

$$\text{Circumference} = 2 \times \pi \times \text{radius}$$

$$C = 2\pi r$$

- If you know the diameter, use this formula:

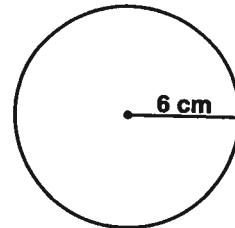
$$\text{Circumference} = \pi \times \text{diameter}$$

$$C = \pi d$$

To find the *area of a circle*, use this formula:

$$\text{Area} = \pi \times \text{radius}^2$$

$$A = \pi r^2$$



$$\text{Circumference} = 2 \times \pi \times r$$

$$C = 2 \times \pi \times 6$$

$$\approx 37.7 \text{ cm}$$

$$\text{Area} = \pi \times r^2$$

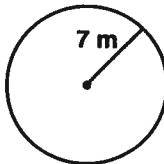
$$A = \pi \times 6^2$$

$$\approx 113.1 \text{ cm}^2$$

The circumference of the circle is about 37.7 cm. The area of the circle is about 113.1 cm^2 .

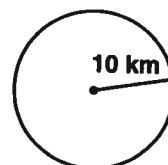
Find the circumference and area of each circle. Round to the nearest tenth.

1.



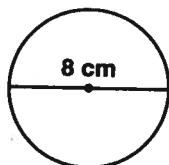
$$C \approx 44.0 \text{ m} \quad A \approx 153.9 \text{ m}^2$$

2.



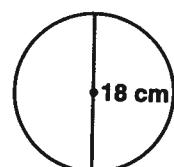
$$C \approx 62.8 \text{ km} \quad A \approx 314 \text{ km}^2$$

3.



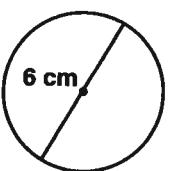
$$C \approx 25.1 \text{ cm} \quad A \approx 50.2 \text{ cm}^2$$

4.



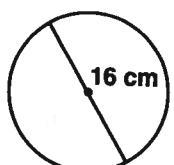
$$C \approx 56.5 \text{ cm} \quad A \approx 254.3 \text{ cm}^2$$

5.



$$C \approx 18.8 \text{ cm} \quad A \approx 28.3 \text{ cm}^2$$

6.



$$C \approx 50.2 \text{ cm} \quad A \approx 201.0 \text{ cm}^2$$

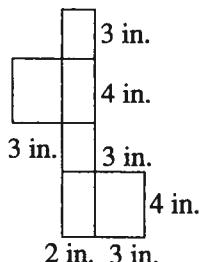
Review 280**Surface Areas of Prisms and Cylinders**

The *surface area* of a solid is the sum of the areas of its surfaces. S.A. stands for *surface area* and L.A. stand for *lateral area*.

Example 1: Find the surface area of the prism.

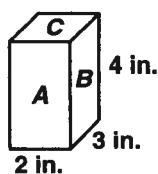
Using a Net to Find Surface Area of a Prism

Draw a net of the prism and find the area of each rectangle in the net.



S.A. =

$$\begin{aligned} & (2 \cdot 3) + (2 \cdot 3) + (3 \cdot 4) + (3 \cdot 4) + (2 \cdot 4) + (2 \cdot 4) \\ & = 6 + 6 + 12 + 12 + 8 + 8 \\ & = 52 \text{ in.}^2 \end{aligned}$$

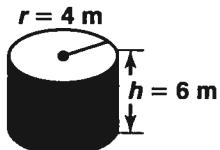
**Using the Prism Surface Area Formula**

The lateral area of a prism is the product of the perimeter of the base and the height of the prism.

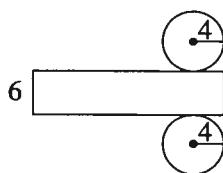
$$\text{L.A.} = ph$$

$$\begin{aligned} \text{S.A.} &= \text{L.A.} + 2B \\ &= ph + 2B \\ &= (2 + 2 + 3 + 3)4 + 2(2 \cdot 3) \\ &= 10(4) + 2(6) \\ &= 40 + 12 = 52 \text{ in.}^2 \end{aligned}$$

Example 2: Find the surface area of the cylinder.

**Using a Net to Find Surface Area of a Cylinder**

Draw a net of the cylinder and find the area of each shape in the net.

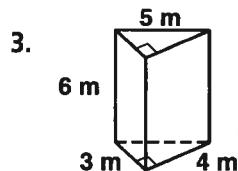
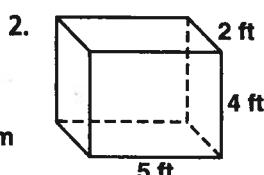
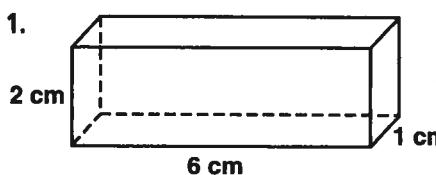


$$\begin{aligned} \text{S.A.} &= 16\pi + 16\pi + 48\pi \\ &= 80\pi \\ &\approx 251.33 \end{aligned}$$

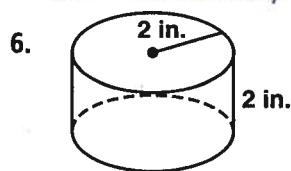
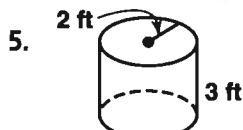
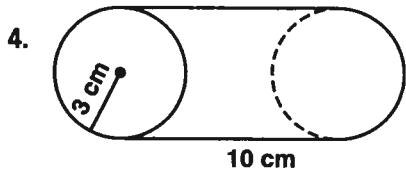
Using the Cylinder Surface Area Formula

$$\begin{aligned} \text{S.A.} &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(4)(6) + 2\pi(4^2) \\ &= 48\pi + 32\pi \\ &= 80\pi \\ &\approx 251.33 \end{aligned}$$

Find the lateral and surface area of each figure to the nearest whole unit.



$$\text{LA} = 28 \text{ cm}^2 \quad \text{SA} = 40 \text{ cm}^2 \quad \text{LA} = 56 \text{ cm}^2 \quad \text{SA} = 76 \text{ cm}^2 \quad \text{LA} = 72 \text{ m}^2 \quad \text{SA} = 84 \text{ m}^2$$



$$\text{LA} = 188 \text{ cm}^2 \quad \text{SA} = 245 \text{ cm}^2 \quad \text{LA} = 38 \text{ ft}^2 \quad \text{SA} = 63 \text{ ft}^2 \quad \text{LA} = 25 \text{ in.}^2 \quad \text{SA} = 50 \text{ in.}^2$$

Review 281**Surface Areas of Pyramids and Cones**

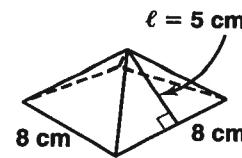
Example 1: Find the surface area of the prism.

The lateral area of a square pyramid is four times the area of one of the lateral faces.

$$\text{L.A.} = 4 \cdot \left(\frac{1}{2}b\ell\right) = 2b\ell$$

The surface area of a square pyramid is the sum of the lateral area and the area of the base.

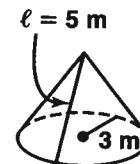
$$\begin{aligned}\text{S.A.} &= \text{L.A.} + B \\ &= 2b\ell + b^2 \\ &= 2(8)(5) + 8^2 \\ &= 80 + 64 \\ &= 144 \text{ cm}^2\end{aligned}$$



Example 2: Find the surface area of the cone.

The lateral area of a cone is one half the product of the circumference of the base and the slant height.

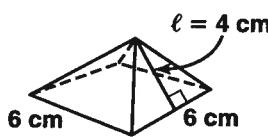
$$\text{L.A.} = \frac{1}{2}(2\pi r)\ell = \pi r\ell$$



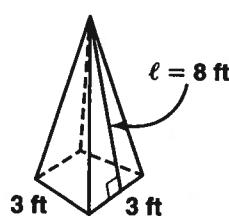
$$\begin{aligned}\text{S.A.} &= \text{L.A.} + B \\ &= \pi r\ell + \pi r^2 \\ &= \pi(3)(5) + \pi(3^2) \\ &= 15\pi + 9\pi \\ &= 24\pi \approx 75.4 \text{ m}^2\end{aligned}$$

Find the lateral and surface area of each square pyramid.

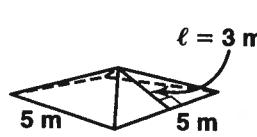
1.



2.



3.



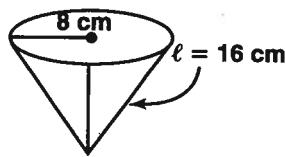
$$\text{LA} = 48 \text{ cm}^2 \quad \text{SA} = 84 \text{ cm}^2$$

$$\text{LA} = 48 \text{ ft}^2 \quad \text{SA} = 57 \text{ ft}^2$$

$$\text{LA} = 30 \text{ ft}^2 \quad \text{SA} = 55 \text{ ft}^2$$

Find the surface area of each cone to the nearest whole unit.

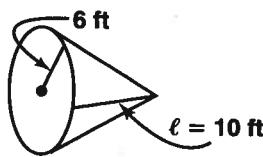
4.



$$\text{SA} = 64\pi + 128\pi$$

$$\text{SA} = 603 \text{ cm}^2$$

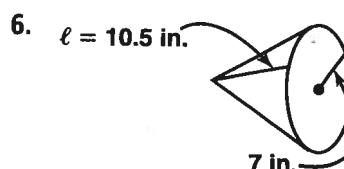
5.



$$\text{SA} = 36\pi + 60\pi$$

$$\text{SA} = 301 \text{ ft}^2$$

6.

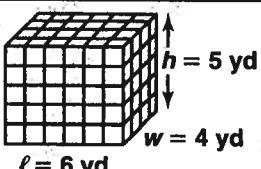
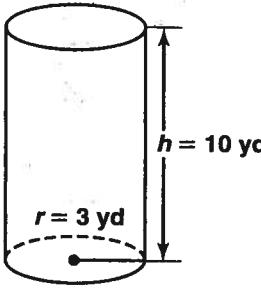


$$\text{SA} = 49\pi + 73.5\pi$$

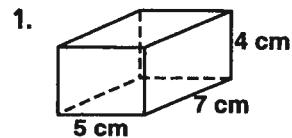
$$\text{SA} = 385 \text{ in}^2$$

Review 282**Volumes of Prisms and Cylinders**

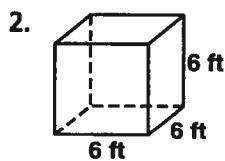
To find the volume of a prism or a cylinder, multiply the base area B and the height h .

	① Find the base area B .	② Multiply base area B and height h . $V = Bh$
	$\begin{aligned}B &= \ell w \\&= 6 \cdot 4 \\&= 24 \text{ yd}^2\end{aligned}$	$\begin{aligned}V &= Bh \\&= 24 \cdot 5 \\&= 120 \text{ yd}^3\end{aligned}$ <p>The volume is 120 yd^3.</p>
	$\begin{aligned}B &= \pi r^2 \\&= \pi \cdot 3^2 \\&\approx 28.27 \text{ yd}^2\end{aligned}$	$\begin{aligned}V &= Bh \\&\approx 28.27 \cdot 10 \\&\approx 282.7 \text{ yd}^3\end{aligned}$ <p>The volume is about 283 yd^3.</p>

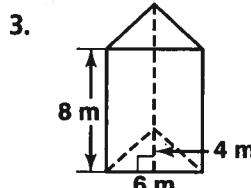
Find the base area and volume of each prism.



$$\begin{aligned}B &= 35 \text{ cm}^2 \\V &= 140 \text{ cm}^3\end{aligned}$$

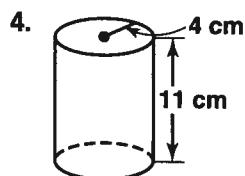


$$\begin{aligned}B &= 36 \text{ ft}^2 \\V &= 216 \text{ ft}^3\end{aligned}$$

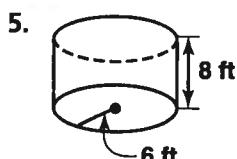


$$\begin{aligned}B &= 12 \text{ m}^2 \\V &= 96 \text{ m}^3\end{aligned}$$

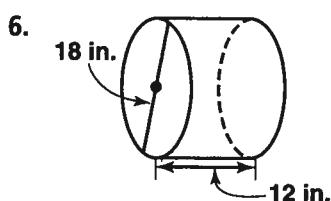
Find the base area of each cylinder to the nearest hundredth. Then find the volume of each cylinder to the nearest whole unit.



$$\begin{aligned}B &\approx 50.24 \text{ cm}^2 \\V &\approx 553 \text{ cm}^3\end{aligned}$$



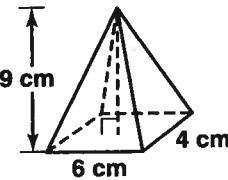
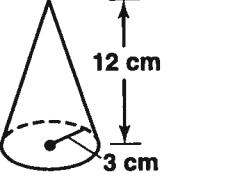
$$\begin{aligned}B &\approx 113.04 \text{ cm}^2 \\V &\approx 904 \text{ cm}^3\end{aligned}$$



$$\begin{aligned}B &\approx 254.34 \text{ in}^2 \\V &\approx 3052 \text{ in}^3\end{aligned}$$

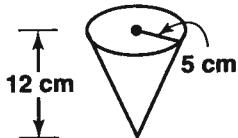
Review 283**Volumes of Pyramids and Cones**

To find the volume of a pyramid or cone, multiply $\frac{1}{3}$, the base area B , and the height h .

① Find the base area B .	② Multiply $\frac{1}{3}$, the base area B , and the height h .
 $B = \ell w$ $= 6 \cdot 4$ $= 24 \text{ cm}^2$	$V = \frac{1}{3}Bh$ $= \frac{1}{3}(24)(9)$ $= 72 \text{ cm}^3$ <p>The volume is 72 cm^3.</p>
 $B = \pi r^2$ $= \pi \cdot 3^2$ $\approx 28.27 \text{ cm}^2$	$V = \frac{1}{3}Bh$ $\approx \frac{1}{3}(28.27)(12)$ $\approx 113.08 \text{ cm}^3$ <p>The volume is about 113.08 cm^3.</p>

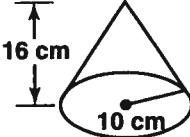
Find the volume of each figure to the nearest whole unit.

1.



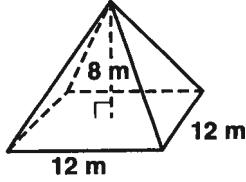
$$V = 314 \text{ cm}^3$$

2.



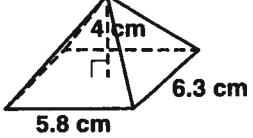
$$V = 1675 \text{ cm}^3$$

3.



$$384 \text{ m}^3$$

4.



$$49 \text{ cm}^3$$

5. Find the height of a cone with an approximate volume of 134 cm^3 and a radius of 4 cm.

$$8 \text{ cm}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$134 = \frac{1}{3} \pi (16) h$$

$$8 = h$$

